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DEVELOPMENT OF A METHOD OF SIMULTANEOUS DETERMINATION OF THE CONDUCTIVITY AND THERMAL DIFFUSIVITY OF MATERIALS

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ABSTRACT

There are several methods of measures of thermo physical properties. The methods of transient state are used to determine the thermal diffusivity and those of steady state who used to determine the thermal conductivity. Most of the devices sold commercially allow measuring these properties on separate tests.

In this paper we have developed a method that allows measuring simultaneously the thermal diffusivity and the thermal conductivity in a single operation. For that we build a very simple device.

This new methods is obtained by solving the transient heat diffusion equation with the method of Duhamel leading to expressions of diffusivity and conductivity for understanding how to perform measurements in transient and permanent to measure these parameters.

The validation of the theoretical model was made from two reference materials: plaster and extruded polystyrene.

KEYWORDS: characterization, thermal conductivity, thermal diffusivity, plaster, extruded polystyrene.

Symbols

a	Thermal diffusivity, m2/s
Bi	Biot number
c	Specific heat, J/kgK
Fo	Fourier number
h	Coefficient convective-exchange, W/m².K
L	Thickness, m
t	time variable, s
T	Temperature, K
u	Reduced variable space
x	variable espace, m
ρ	density, kg/m3
λ	Thermal conductivity, W/mK
τ	Reduced variable time
δ	difference
θ	Reduced temperature
ω	Own value reduced
ex	Outside
exp	Experimental
do	Decreasing
A	Ambient
B	Box
C	Hot face
F	Cold face
1	Not irradiated faced
2	Irradieted face
g	Overall

INTRODUCTION

In Senegal, as in most African countries, lot of experimental studies are made on new building materials for economic reasons and for the thermal comfort in the home.

Despite the abundance of published work, it remains lot of materials which properties are unknown [1], [2]. The knowledge of thermophysical properties being essential for their optimum use and devices to measure are difficult, that is why we have developed a simple device easy to build.

So we give the method of determining the desired properties by giving a model resolution of the transient heat diffusion equation. We also give the experimental study and results.

MATERIALS AND METHODS

Modeling

Consider a sample of constant thickness L that is small compared to the dimensions of the length and width and heated on its upper face.

Assuming that:

- The heat conduction is mono dimensional.
- The parameters ***ρ***, ***c***, ***λ*** and ***a*** are independent of temperature.

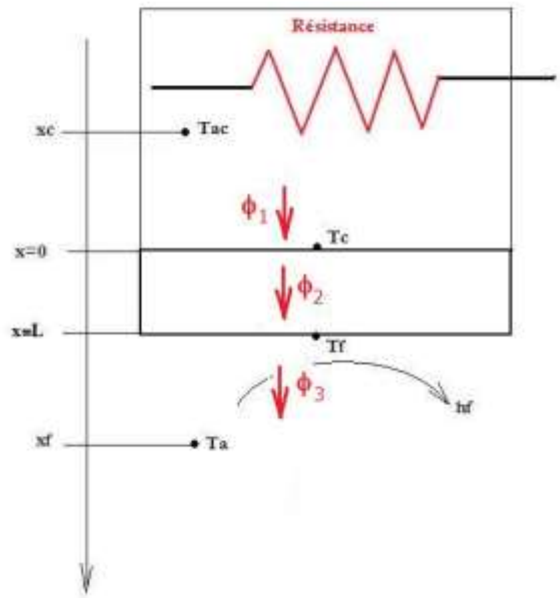

Figure 1: Schematic model

The thermal field [3], [4], [5] in this sample is governed by the equation:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{a}\frac{\partial T(x,t)}{\partial t} \qquad (1)$$

With boundary conditions:

- At the front face (x=0) :

$$\lambda \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = RI^2 - C(Tac - Ta) \qquad (2)$$

- At the rear face (x=L) :

$$\lambda \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=L} = hf(Tf - Ta) \qquad (3)$$

where:

- $T_{ac}(t)$: room temperature listed above depending on the parameter t,
- T_f : temperature of the underside,
- T_a : room temperature low side , constant,
- R : heating resistor,
- I : current flowing through the resistor,
- C : Loss coefficient of the box.

Note that the boundary conditions are time functions Tac(t) et Tf(t).

To solve the equation (1) taking into account these conditions using the theorem Duhamel [3] [4] [6] introducing $\Theta(x,t,t_1)$ as auxiliary solution.

$\Theta(x,t,t_1)$ satisfies the equation (1) and the boundary conditions (2) and (3).

Then:

$$\frac{\partial^2 \Theta_i(x,t,t_1)}{\partial x^2} = \frac{1}{a}\frac{\partial \Theta_i(x,t,t_1)}{\partial t} \qquad (4)$$

With boundary conditions:

$$\left.\frac{\partial \Theta(x,t,t_1)}{\partial x}\right|_{x=0} = \frac{1}{\lambda}\left[RI^2 - C(\Theta_i(0,t,t_1) - T_a)\right] \qquad (5)$$

$$\left.\frac{\partial \Theta_i(x,t,t_1)}{\partial x}\right|_{x=L} = \frac{h_f}{\lambda}\left[\Theta_i(L,t,t_1) - T_a\right] \qquad (6)$$

Note that T_a does not depend on time t, but rather the constant parameter t_1 (t_1 is not a time variable).

The solution which gives T (x, t) of equation (1) is of the form:

$$T_i(x,t) = \int_{t_1=0}^{t} \frac{\partial}{\partial t}\left[\Theta(x,t-t_1,t_1)\right]dt_1 \qquad (7)$$

The differential equations (5) and (6) are not homogeneous because of the existence of a constant term independent of variable Θ.

To simplify the solution of (4), we rewrite the equations (4), (5) and (6) reduced in dimensionless form by introducing a reduced space variable:

$$u = \frac{x}{L} = \frac{x}{e} \qquad (8)$$

And reduced temperature variable

$$\theta(u,\tau,\tau_1) = \frac{\Theta_i(x,t,t_1) - T_a}{T_a} \qquad (9)$$

Where τ_1 is the reduced parameter corresponding to t_1.
Substituting u and θ in the differential equations (4), (5) and (6), it is the equation of heat conduction:

$$\frac{\partial^2 \theta_i(u,\tau,\tau_1)}{\partial u^2} = \frac{\partial \theta_i(u,\tau,\tau_1)}{\partial \tau} \qquad (10)$$

With the boundary conditions:

$$\left.\frac{\partial \theta_i(u,\tau,\tau_1)}{\partial u}\right|_{u=0} = \frac{RI^2}{\lambda} - \frac{C}{\lambda}\left[\Theta_i(0,\tau,\tau_1) - T_a\right] \qquad (11)$$

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$$\left.\frac{\partial\theta_i(u,\tau,\tau_1)}{\partial u}\right|_{u=1} = \frac{h_f}{\lambda}\left[\Theta_i(1,\tau,\tau_1)-T_a\right] \qquad (12)$$

Where τ is a reduced variable time or number of Fourier

$$\tau = \frac{at}{L^2} \equiv F_0 \qquad (13)$$

The reduced temperature of the sample θ (u, τ, τ1) can be written with a reduced time τ :

$$\theta_i(u,\tau,\tau_1) = \theta_i(u,\tau_1) + \delta\theta_i(u,\tau,\tau_1) \qquad (14)$$

With $\theta_i(u,\tau_1)$: reduced temperature corresponding to steady and;

$\delta\theta_i(u,\tau,\tau_1)$: reduced temperature corresponding to the transitional regime.

Substituting expression (14) in equations (10), (11) and (12) and separating these equations in two groups governing reduced temperatures corresponding to the steady state and transient regimes. The system transient is described by:

$$\frac{\partial^2\delta\theta_i(u,\tau,\tau_1)}{\partial u^2} = \frac{\partial\delta\theta_i(u,\tau,\tau_1)}{\partial\tau} \qquad (15)$$

With the boundary conditions:

$$\left.\frac{\partial\delta\theta_i(u,\tau,\tau_1)}{\partial u}\right|_{u=0} = -\frac{C}{\lambda}\delta\Theta_i(0,\tau,\tau_1) \qquad (16)$$

$$\left.\frac{\partial\delta\theta_i(u,\tau,\tau_1)}{\partial u}\right|_{u=1} = \frac{h_f}{\lambda}\delta\Theta_i(1,\tau,\tau_1) \qquad (17)$$

The study on transient state gives the solution of the temperature field in the sample. We check a solution in form of separable variable. It comes:

$$\delta\theta_i(u,\tau,\tau_1) = U_i(u,\tau_1).T_i(\tau) \qquad (18)$$

Returning to equation (15) we obtain:

$$\frac{1}{U_i(u,\tau_1)}.\frac{\partial^2 U_i(u,\tau_1)}{\partial u^2} = \frac{1}{T_i(\tau)}.\frac{\partial T_i(\tau)}{\partial\tau} = -\omega_i^{\ 2} \qquad (19)$$

The solution of equation (19) is on the form ($\omega_n > 0$ and n Є N) :

$$T_{in}(\tau) = T_{in}(0)\exp\left[-(\omega_{in})^2\tau\right] = T_{in}(0)\exp\left[-\frac{\tau}{\tau_{do}}\right] \qquad (20)$$

Where ω_i is a positive self-worth.

Using the boundary conditions, the transcendental equation is determined (21) for determining the parameters ω_i: graphics solution (Figure 2), the intersection of the two curves.

$$\tan(\omega_n) = -\frac{Bi}{L}.\frac{1}{\omega_n} \qquad (21)$$

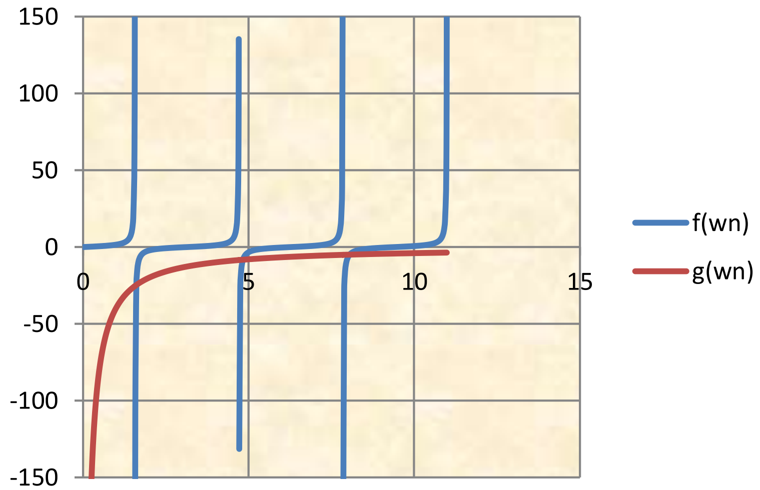

Figure 2: Plot of the transcendental equation

Table 1: graphical solutions of the transcendental equation:

Valeur de ω_n	ω_1	ω_2	ω_3
	1.39	4.62	7.86

From equation (7) we obtain:

$$\Theta(x,t,t_1) = T_a\left[1 + \theta_i(u,\tau_1) + \delta\theta_i(u,\tau,\tau_1)\right] \qquad (22)$$

After integration, the expression of T (x, t) is:

$$T_i(x,t) = \frac{T_a U}{t_{do}}\left[1 - \exp\left(-\frac{t}{t_{do}}\right)\right] \qquad (23)$$

With:

$$U = \frac{2\delta\theta_o}{\sin^2(\omega_{in})}\left[\frac{1}{2} + \frac{\sin(2\omega_{in})}{4\omega_{in}}\right] \qquad (24)$$

The decay time constant t_{do} is determined from the relaxation curve of the experimental temperature Texp(x,t).

Determining the thermal diffusivity

The decay time constant $\tau_{do} = \dfrac{1}{\omega_n^2}$ gives the expressions for short times that can be inferred from the decay time constant and give the expression for the thermal diffusivity [7]:

$$a = \frac{b.L^2}{\omega_{in}^2} \quad (25)$$

Determining the thermal conductivity

The heat flux through the sample is given by the equation (26). This stream is produced by an electric current V through a resistor R. It is given by the equation (27). We made a heat balance [8], [9] to obtain the thermal conductivity λ :

$$\Phi = \frac{\lambda}{L}(T_C - T_F) \times A + C(T_B - T_a) \quad (26)$$

$$\Phi = \frac{V^2}{R} \quad (27)$$

$$\lambda = \frac{L}{A.(T_C - T_F)}\left[\frac{V^2}{R} - C(T_B - T_a)\right] \quad (28)$$

EXPERIENCE

Experimental device

The theoretical study of the model with well-studied conditions limits gave us mathematical expressions that can be interpreted to exit the parameters that allow us to determine thermo physical characteristics of a given material.

It is noted that for determining the diffusivity (a) we essentially need the slope of the regression curve of the temperature of the unheated surface. Which is easily done with a thermocouple stuck to the cold face (figure 3). For conductivity, steady state is required to have correct values.

The experience also allowed us to determine the best conditions for each type of measurement.

The constructed apparatus is shown in Figure 3 and 4. One of the faces of the sample receives the heat of the heating film and the other is in contact with ambient air in a controlled temperature room (air-conditioned) [10], [11].

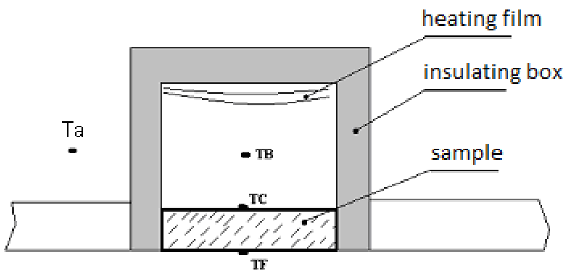

Figure 3: the sample position in the box

For fixing the box and the sample, we conducted a metal support made of 30mm×30mm bracket and feet welded square tubes. The system thus assembled is illustrated in Figures 4.

Figure 4: measuring device

Determination of sample characteristics parameters

We studied mainly two materials: plaster and polystyrene. Both materials are simple and known materials [10] [11]. It is an acquisition temperature at different points of the sample. (TC: hot face temperature TB: temperature of the box, TF: cold face temperature Ta: ambient temperature of the room).

Measurement is launched on a very long time (more than two hours), acquisitions made every 60 s with a data logger controlled by a computer with an Agilent application.

For plaster

The curves of changes in temperatures (Figure 5) show that mainly two parts: the one where there's a change in temperature and a other where temperatures change more.

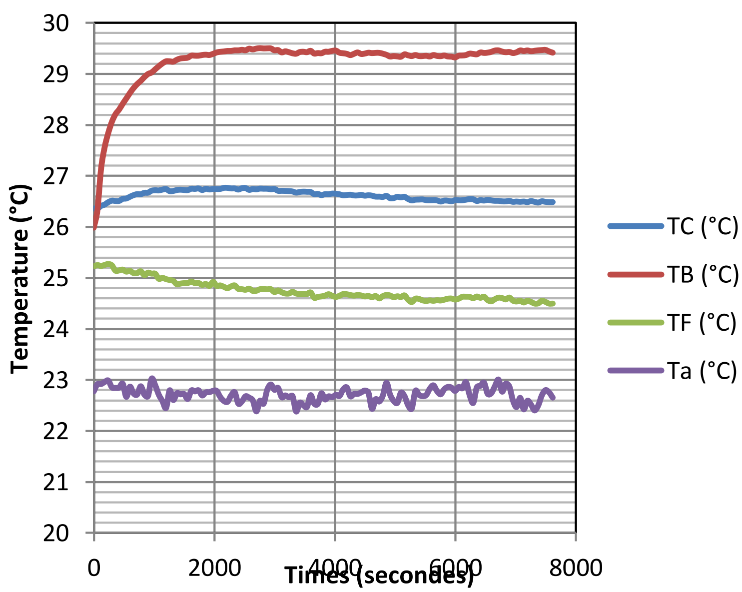

Figure 5: Evolutions temperature at various points, V=100V

There is a rapid increase in temperature (TB) of the box for 30 minutes and stabilizes around 29.5°C. For the temperature of the heated surface (hot surface), TC shows the same phenomenon, but its temperature was stabilized at around 26.5°C. For the cold surface (TF), there is a temperature drop and then stabilize around 24.5°C. The ambient temperature remained constant (air conditioned room ≈ 23 ° C).

The steady state is achieved after 45 minutes.

For the value of the diffusivity, an average is given.

Then we plot the regression curve of the change of the temperature of the cold face on figure 6.

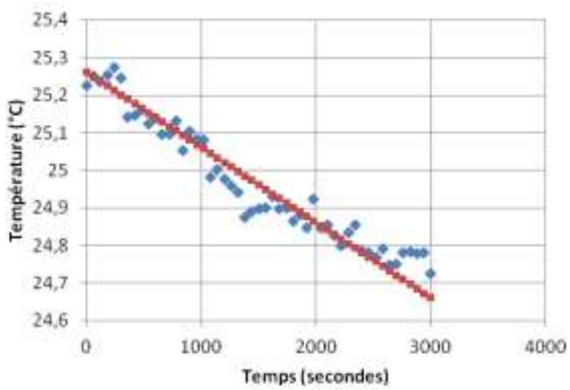

Figure 6: Regression curve of the temperature of the cold face

For polystyrene

We made an acquisition of temperatures at various points in our sample. Measurement is has taken more than two hours with an acquisition every 60s. Changes in temperatures of different points give the following curves.

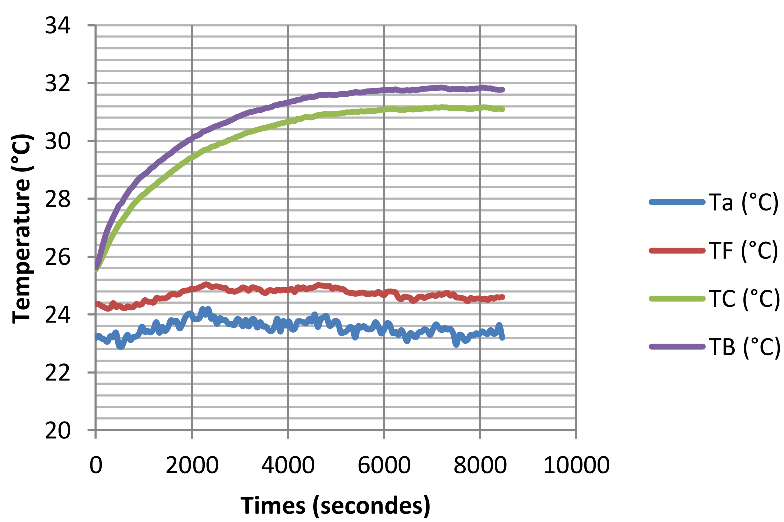

Figure 7: Evolutions temperatures at various points, V=100V

We are seeing a general increase in temperatures, at time t = 400 seconds the increase was slower and temperatures stabilize. The operation of these temperature evolution curves gives a regression line of the cold face:

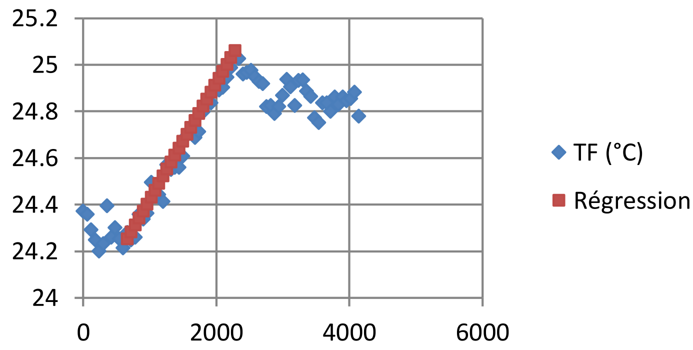

Figure 8: regression curve of the temperature of the cold face

RESULTS AND DISCUSSION

Results

Table 2: Experimental results

Material	Voltage V	Measuring time	Conductivity λ ($Wm^{-1}K^{-1}$)
Plaster	220 V	3 hours 30 mn	*0.265*
Plaster	220 V	3 hours	*0.244*
Plaster	200 V	2 hours 30 mn	*0.254*

The average conductivity of the sample is: 0.25 $W.m^{-1}.K^{-1}$ with a density of 972 $kg.m^{-3}$. This result can be retained with an uncertainty of 7%.

Table 3: Experimental results

Material	Voltage V	Measuring time	Diffusivity a (m^2s^{-1})
Plaster	220 V	1 hour 40 mn	*5.17×10^{-7}*
Plaster	220 V	1 hour 45 mn	*2.59×10^{-6}*
Plaster	200 V	55 minutes	*3.88×10^{-7}*

It is observed in these three cases an uniform increase in the temperature of the cold face. These three cases give us acceptable results. We get an average diffusivity a = $3.88 \times 10^{-7} m^2s^{-1}$ with a relative uncertainty of 13%.

The same experience was repeated with expanded polystyrene and we obtained the following results:

Table 4: Experimental results

Material	Voltage V	Measuring time	Conductivity λ ($Wm^{-1}K^{-1}$)
extruded polystyrene	100 V	1 hour 45 mn	*0.03241*
extruded polystyrene	100 V	2 hours 15mn	*0.0240*

The average conductivity of the sample is: 0.023W / m K. This can be selected with a relative uncertainty of 20%.

Table 5: Experimental results

Material	Voltage V	Measuring time	Diffusivity a (m^2s^{-1})
extruded polystyrene	100 V	50 mn	*6.47 x 10^{-7}*
extruded polystyrene	200 V	45 mn	*7.76 x 10^{-7}*

The mean diffusivity in our sample is *7,115x10^{-7}*m^2s^{-1}. This can be selected with a relative uncertainty of 14%.

These results are acceptable because they are closed to those obtained with a widely used method. For example with DEGIOVANNI method give a result for the extruded polystyrene a diffusivity of 7.84 *x*10^{-7}m^2s^{-1} [8] [9].

CONCLUSION

A method of determining thermal diffusivity and conductivity has been developed in this work. The theoretical study gave, with defined starting conditions, an analytical solution of the temperature changes we obtained using methods already established namely, the theory of DUHAMEL.

Different results have given the solution that allows us to have the thermal diffusivity by exploiting the curve of the change in temperature. The experimental curve provides a slope, which is used for determining the diffusivity. The conductivity is obtained when the steady state is reached. This theoretical study and boundary conditions used made the experimental device light. We experimented on a sample size and obtained for the plaster the following results:

- Diffusivity : a=3.88 *x* $10^{-7}m^2s^{-1}$
- Conductivity : λ = 0.25 $Wm^{-1}K^{-1}$

In a second experiment with an extruded polystyrene sample and the same measurement conditions are obtained:

- Diffusivity : a=7.115 *x* $10^{-7}m^2s^{-1}$
- Conductivity : λ=0.023$Wm^{-1}K^{-1}$

These results are close to the values given in the literature.

This technique is not expensive, and can be used to make an estimate of the thermo physical coefficients of building construction materials. The study of thermal comfort in buildings can thus be best achieved if we have the values of diffusivity and conductivity of all material prior to use. In Senegal, there is not yet a thermal regulation as in developed countries. So this technique can help to characterize rapidly materials with at low cost.

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